Shpaner, Leon

Applied Time Series : Module 1 Presentation

ch 1. → models

ch 2. → Stationarity

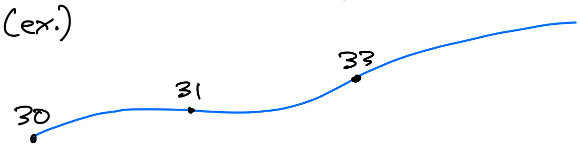
auto correlation

auto covariance

customer	What	$	

Time Series - measuring something that changes over time, or sampling a signal / measuring same signal, but multiple times

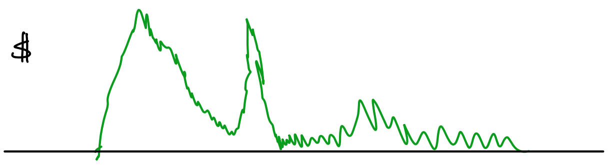
temp (ex.)



index 10am 11am 12pm

↳ becomes column (structure to capture the signal)

(ex.) Stock Market / financial data

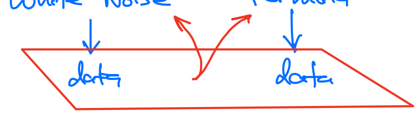


Algorithms vs. Models

Algorithm ↳ ML Model → Prediction

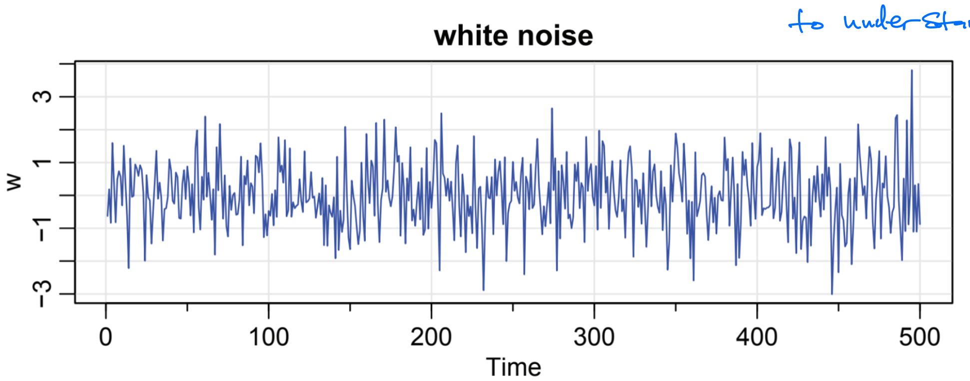
Time Series Models ↳ inductive reasoning

White Noise Formula



inductive reasoning

We only have the data, and want to go back to understand how the world works.



White Noise

$W_t \sim w_n(0, \sigma_w^2)$

index in array → mean → standard deviation, w

Appendix B has good stats review

Weight at time sub t is coming from some probability distribution, and you have some mean, and you have some variance. Nothing on the right-hand side depends on t.

Every time we take some sample, it just generates some random number

The time points have nothing to do w/ each other → most boring kind of time series.

↳ next page

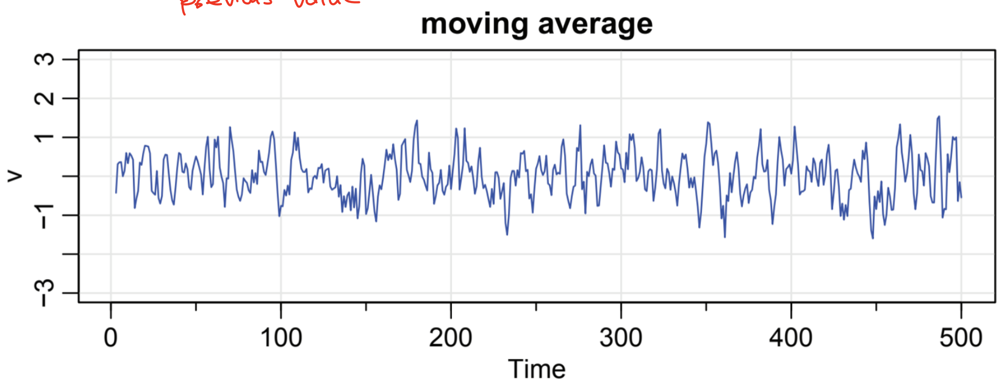
Smoothing – Moving Average

$$v_t = \frac{1}{3}\left(w_{t-1} + w_t + w_{t+1}\right)$$

→ calculated from the white noise

previous value

→ next value



run t across whole signal in a for-loop. Notice how moving average is losing jaggedness of top white noise plot, and is smaller in amplitude.

Autoregression → Similar to linear regression

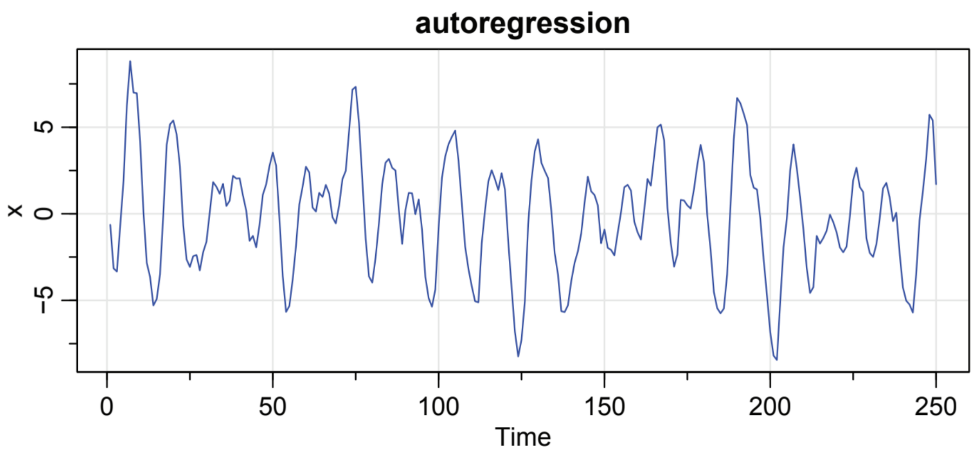
auto b/c being predicted by itself

Where it's going to go next depends on where it was a little bit ago.

$$x_t = 1.5x_{t-1} - .75x_{t-2} + w_t$$

white noise

- Can write a for-loop to keep pushing fresh noise to the model, and it will look something like this:

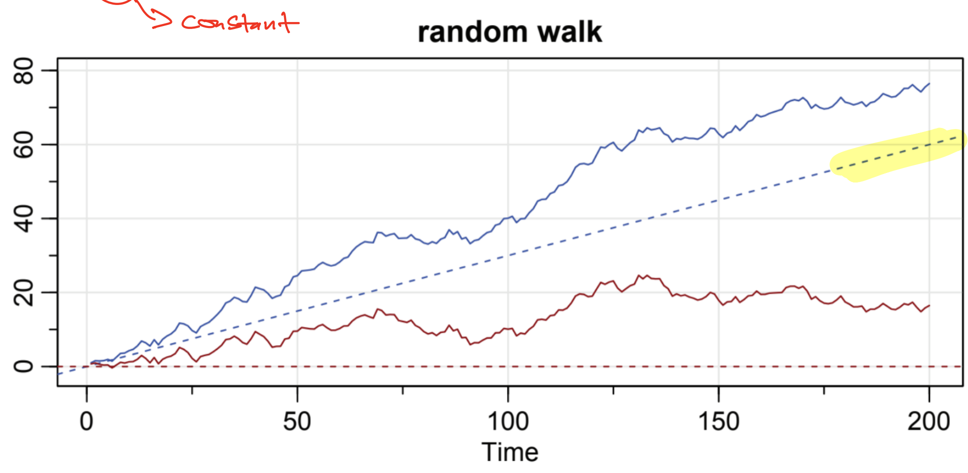


→ less jagged, more smooth than previous ones. The future, here, depends on the past in a really strong way.

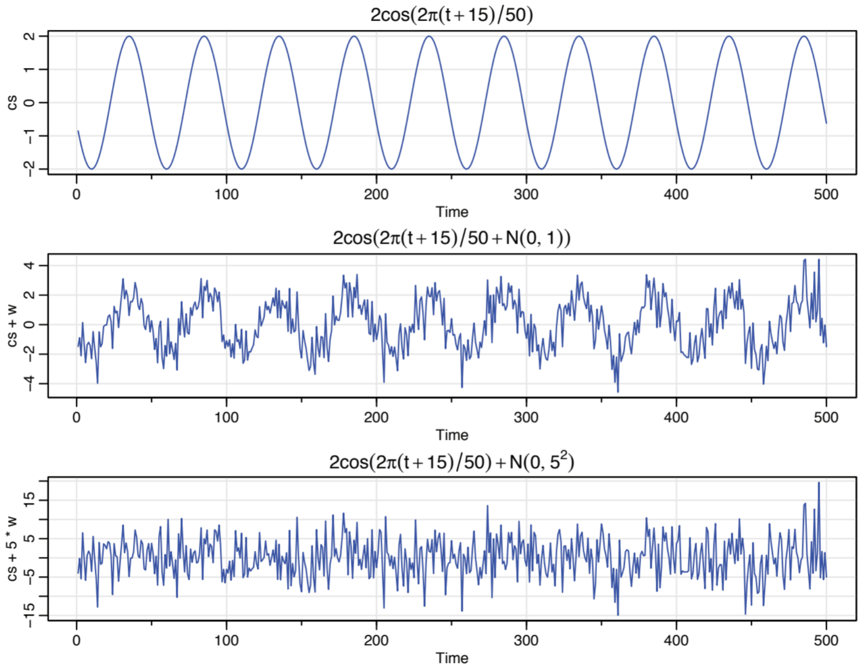
Random Walk w/ Drift → ex. how particles move around the room.

$$x_t = \delta + w_{t-1} + w_t$$

→ constant



→ drift



Chapter 2

Mean ~ Expectation

$\int x\, f(x)$ → if normal distribution, integrate over domain.

$\sum x\, f(x)$ → if discrete, replace w/ sum over all of the possibilities.

ex. Dice: $1(1/6) + 2(1/6) + 3(1/6) + \cdots + n(1/6) = 2\frac{1}{2}$ → measure of central tendency

Linearity Expectation

$$E(a + bx) = a + bE(x)$$

$$E(x) = \mu_x$$

Variance $E(x-\mu)^2 = \int (x-\mu)^2 f(x)\,dx$

Covariance

$$\mathrm{cov}(x,y) = E(x-\mu_x)(y-\mu_y)$$

covariance w/ itself is variance

Correlation

$\{-1, 1\}$

$$\frac{\delta_{xy}}{\delta_x \delta_y}$$

Covariance (Mean Function)

$$E[(x_s - \mu_s)(x_t - \mu_t)] = \gamma(s,t) \rightarrow \text{autocovariance}$$

time mean time

autocorrelation

$$\frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\,\gamma(t,t)}}$$

mean function

$$E(X_t) = \mu_{x_t}$$

$$E(V_t) = E\left(\frac{1}{3} w_{t-1} + \frac{1}{3} w_t + \frac{1}{3} w_{t+1}\right)$$

$$= \frac{1}{3} E(w_{t-1}) + \frac{1}{3} E(w_t) + \frac{1}{3} E(w_{t+1})$$

$$= \frac{1}{3}(0) + \frac{1}{3}(0) + \frac{1}{3}(0)$$

$$= 0$$

Random walk

$$X_t = \delta_t - 1 \sum_{j=1}^{t} w_j$$

$$= E(\delta_t) + E\left(\cancel{\sum_{j=1}^{t} w_j}\right)$$

$$E(x_t) = \delta_t$$

Stationarity

μ_x – constant, not depend on time

$\gamma(s,t)$ – only depends on time difference

ex.

$\gamma(4,1) = \gamma(5,2)$ where lag $= 3 \Rightarrow$ same

ex. White noise

Random walk model is <u>not</u> Stationary b/c the mean depends on time.

$$\text{Sample ACF} = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} = \frac{\overbrace{(x_{t+h} - \bar{x})(x_t - \bar{x})}^{\text{cov}}}{\sum \underbrace{(x_t - \bar{x})^2}_{\text{variance}}}$$